The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Tuesday, June 15, 2010—9:15 a.m. to 12:15 p.m., only

Student Name: ______________________________________________________________

School Name: _______________________________________________________________

Print your name and the name of your school on the lines above.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions, using a No. 2 pencil, on the separate answer sheet provided to you. Write your answers to the questions in Parts II, III, and IV directly in this test booklet. All work for Parts II, III, and IV should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…
A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, record your answer, using a No. 2 pencil, on the separate answer sheet provided to you. [54]

1. What is the common difference of the arithmetic sequence 5, 8, 11, 14?
   - (1) \( \frac{3}{5} \)
   - (2) \(-3\)
   - (3) 3
   - (4) 9

2. What is the number of degrees in an angle whose radian measure is \( \frac{11\pi}{12} \)?
   - (1) 150
   - (2) 165
   - (3) 330
   - (4) 518

3. If \( a = 3 \) and \( b = -2 \), what is the value of the expression \( \frac{a-2}{b-3} \)?
   - (1) \( -\frac{9}{8} \)
   - (2) \(-1\)
   - (3) \( -\frac{8}{9} \)
   - (4) \( \frac{8}{9} \)
4. Four points on the graph of the function \( f(x) \) are shown below. \((0,1), (1,2), (2,4), (3,8)\)

Which equation represents \( f(x) \)?

1. \( f(x) = 2^x \)
2. \( f(x) = 2x \)
3. \( f(x) = x + 1 \)
4. \( f(x) = \log_2 x \)

5. The graph of \( y = f(x) \) is shown below.

Which set lists all the real solutions of \( f(x) = 0 \)?

1. \( \{-3, 2\} \)
2. \( \{-2, 3\} \)
3. \( \{-3, 0, 2\} \)
4. \( \{-2, 0, 3\} \)
6 In simplest form, $\sqrt{-300}$ is equivalent to

(1) $3i\sqrt{10}$  (3) $10i\sqrt{3}$
(2) $5i\sqrt{12}$  (4) $12i\sqrt{5}$

7 Twenty different cameras will be assigned to several boxes. Three cameras will be randomly selected and assigned to box A. Which expression can be used to calculate the number of ways that three cameras can be assigned to box A?

(1) $20!$  (3) $20C_3$
(2) $\frac{20!}{3!}$  (4) $20P_3$

8 Factored completely, the expression $12x^4 + 10x^3 - 12x^2$ is equivalent to

(1) $x^2(4x + 6)(3x - 2)$  (3) $2x^2(2x - 3)(3x + 2)$
(2) $2(2x^2 + 3x)(3x^2 - 2x)$  (4) $2x^2(2x + 3)(3x - 2)$

9 The solutions of the equation $y^2 - 3y = 9$ are

(1) $\frac{3 \pm 3i\sqrt{3}}{2}$  (3) $\frac{-3 \pm 3i\sqrt{5}}{2}$
(2) $\frac{3 \pm 3i\sqrt{5}}{2}$  (4) $\frac{3 \pm 3i\sqrt{5}}{2}$
10 The expression $2 \log x - (3 \log y + \log z)$ is equivalent to
   (1) $\log \frac{x^2}{y^3z}$  \hspace{1cm} (3) $\log \frac{2x}{3yz}$
   (2) $\log \frac{x}{y^3}$  \hspace{1cm} (4) $\log \frac{2x}{3y}$

11 The expression $(x^2 - 1)^{-\frac{2}{3}}$ is equivalent to
   (1) $\frac{3}{\sqrt[3]{(x^2 - 1)^2}}$  \hspace{1cm} (3) $\sqrt[3]{(x^2 - 1)^3}$
   (2) $\frac{1}{\sqrt[3]{(x^2 - 1)^2}}$  \hspace{1cm} (4) $\frac{1}{\sqrt[3]{(x^2 - 1)^3}}$

12 Which expression is equivalent to $\frac{\sqrt{3} + 5}{\sqrt{3} - 5}$?
   (1) $-\frac{14 + 5\sqrt{3}}{11}$  \hspace{1cm} (3) $\frac{14 + 5\sqrt{3}}{14}$
   (2) $-\frac{17 + 5\sqrt{3}}{11}$  \hspace{1cm} (4) $\frac{17 + 5\sqrt{3}}{14}$

13 Which relation is not a function?
   (1) $(x - 2)^2 + y^2 = 4$  \hspace{1cm} (3) $x + y = 4$
   (2) $x^2 + 4x + y = 4$  \hspace{1cm} (4) $xy = 4$
14 If $\angle A$ is acute and $\tan A = \frac{2}{3}$, then

(1) $\cot A = \frac{2}{3}$  
(2) $\cot A = \frac{1}{3}$  
(3) $\cot(90^\circ - A) = \frac{2}{3}$  
(4) $\cot(90^\circ - A) = \frac{1}{3}$

15 The solution set of $4x^2 + 4x = 2 - 6$ is

(1) $\{1, 3\}$  
(2) $\{-1, 3\}$  
(3) $\{-1, -3\}$  
(4) $\{1, -3\}$

16 The equation $x^2 + y^2 - 2x + 6y + 3 = 0$ is equivalent to

(1) $(x - 1)^2 + (y + 3)^2 = -3$  
(2) $(x - 1)^2 + (y + 3)^2 = 7$  
(3) $(x + 1)^2 + (y + 3)^2 = 7$  
(4) $(x + 1)^2 + (y + 3)^2 = 10$
17 Which graph best represents the inequality $y + 6 \geq x^2 - x$?

18 The solution set of the equation $\sqrt{x + 3} = 3 - x$ is

(1) [1]  (3) [1, 6]
(2) [0]  (4) [2, 3]
19 The product of $i^7$ and $i^5$ is equivalent to

(1) 1  (3) $i$
(2) $-1$  (4) $-i$

20 Which equation is represented by the graph below?

\[ y = \cot x \quad (3) \ y = \sec x \]
(2) $y = \csc x$  (4) $y = \tan x$

21 Which value of $r$ represents data with a strong negative linear correlation between two variables?

(1) $-1.07$  (3) $-0.14$
(2) $-0.89$  (4) 0.92
22 The function \( f(x) = \tan x \) is defined in such a way that \( f^{-1}(x) \) is a function. What can be the domain of \( f(x) \)?

(1) \( \{ x \mid 0 \leq x \leq \pi \} \)

(2) \( \{ x \mid 0 \leq x \leq 2\pi \} \)

(3) \( \{ x \mid -\frac{\pi}{2} < x < \frac{\pi}{2} \} \)

(4) \( \{ x \mid -\frac{\pi}{2} < x < \frac{3\pi}{2} \} \)

23 In the diagram below of right triangle \( KTW \), \( KW = 6 \), \( KT = 5 \), and \( m\angle KTW = 90 \).

What is the measure of \( \angle K \), to the nearest minute?

(1) 33°33'

(2) 33°34'

(3) 33°55'

(4) 33°56'

24 The expression \( \cos^2 \theta - \cos 2\theta \) is equivalent to

(1) \( \sin^2 \theta \)

(2) \( -\sin^2 \theta \)

(3) \( \cos^2 \theta + 1 \)

(4) \( -\cos^2 \theta - 1 \)
25 Mrs. Hill asked her students to express the sum $1 + 3 + 5 + 7 + 9 + \ldots + 39$ using sigma notation. Four different student answers were given. Which student answer is correct?

(1) $\sum_{k=1}^{20} (2k - 1)$

(2) $\sum_{k=2}^{40} (k - 1)$

(3) $\sum_{k=-1}^{37} (k + 2)$

(4) $\sum_{k=1}^{39} (2k - 1)$

26 What is the formula for the $n$th term of the sequence 54, 18, 6, ...?

(1) $a_n = 6 \left(\frac{1}{3}\right)^n$

(2) $a_n = 6 \left(\frac{1}{3}\right)^{n-1}$

(3) $a_n = 54 \left(\frac{1}{3}\right)^n$

(4) $a_n = 54 \left(\frac{1}{3}\right)^{n-1}$

27 What is the period of the function $y = \frac{1}{2} \sin \left(\frac{x}{3} - \pi\right)$?

(1) $\frac{1}{2}$

(2) $\frac{1}{3}$

(3) $\frac{2}{3} \pi$

(4) $6 \pi$
PART II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

28 Use the discriminant to determine all values of $k$ that would result in the equation $x^2 - kx + 4 = 0$ having equal roots.
The scores of one class on the Unit 2 mathematics test are shown in the table below.

<table>
<thead>
<tr>
<th>Test Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>96</td>
<td>1</td>
</tr>
<tr>
<td>92</td>
<td>2</td>
</tr>
<tr>
<td>84</td>
<td>5</td>
</tr>
<tr>
<td>80</td>
<td>3</td>
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<tr>
<td>76</td>
<td>6</td>
</tr>
<tr>
<td>72</td>
<td>3</td>
</tr>
<tr>
<td>68</td>
<td>2</td>
</tr>
</tbody>
</table>

Find the population standard deviation of these scores, to the nearest tenth.
30 Find the sum and product of the roots of the equation $5x^2 + 11x - 3 = 0$. 
The graph of the equation \( y = \left( \frac{1}{2} \right)^x \) has an asymptote. On the grid below, sketch the graph of \( y = \left( \frac{1}{2} \right)^x \) and write the equation of this asymptote.
32. Express $5\sqrt{3x^3} - 2\sqrt{27x^5}$ in simplest radical form.
33 On the unit circle shown in the diagram below, sketch an angle, in standard position, whose degree measure is $240^\circ$ and find the exact value of $\sin 240^\circ$. 
34 Two sides of a parallelogram are 24 feet and 30 feet. The measure of the angle between these sides is 57°. Find the area of the parallelogram, to the nearest square foot.
35 Express in simplest form:
\[
\frac{\frac{1}{2} - \frac{4}{d}}{\frac{1}{d} + \frac{3}{2d}}
\]
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 The members of a men’s club have a choice of wearing black or red vests to their club meetings. A study done over a period of many years determined that the percentage of black vests worn is 60%. If there are 10 men at a club meeting on a given night, what is the probability, to the nearest thousandth, that at least 8 of the vests worn will be black?
Find all values of $\theta$ in the interval $0^\circ \leq \theta < 360^\circ$ that satisfy the equation $\sin 2\theta = \sin \theta$. 
38 The letters of any word can be rearranged. Carol believes that the number of different 9-letter arrangements of the word “TENNESSEE” is greater than the number of different 7-letter arrangements of the word “VERMONT.” Is she correct? Justify your answer.
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

39 In a triangle, two sides that measure 6 cm and 10 cm form an angle that measures 80°. Find, to the nearest degree, the measure of the smallest angle in the triangle.
Scrap Graph Paper — This sheet will not be scored.
Scrap Graph Paper — This sheet will *not* be scored.
Reference Sheet

Area of a Triangle
\[ K = \frac{1}{2} ab \sin C \]

Functions of the Sum of Two Angles
\[
\begin{align*}
\sin (A + B) &= \sin A \cos B + \cos A \sin B \\
\cos (A + B) &= \cos A \cos B - \sin A \sin B \\
\tan (A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B}
\end{align*}
\]

Functions of the Difference of Two Angles
\[
\begin{align*}
\sin (A - B) &= \sin A \cos B - \cos A \sin B \\
\cos (A - B) &= \cos A \cos B + \sin A \sin B \\
\tan (A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}
\end{align*}
\]

Law of Sines
\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Sum of a Finite Arithmetic Series
\[ S_n = \frac{n(a_1 + a_n)}{2} \]

Binomial Theorem
\[
(a + b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r
\]

Law of Cosines
\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Functions of the Double Angle
\[
\begin{align*}
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
\cos 2A &= 2 \cos^2 A - 1 \\
\cos 2A &= 1 - 2 \sin^2 A \\
\tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}
\end{align*}
\]

Functions of the Half Angle
\[
\begin{align*}
\sin \frac{1}{2} A &= \pm \sqrt{\frac{1 - \cos A}{2}} \\
\cos \frac{1}{2} A &= \pm \sqrt{\frac{1 + \cos A}{2}} \\
\tan \frac{1}{2} A &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}
\end{align*}
\]

Sum of a Finite Geometric Series
\[ S_n = \frac{a_1(1 - r^n)}{1 - r} \]

Normal Curve

Standard Deviation
For handscoring of Part I, this key must be aligned with the answer sheet so that the box indicating the correct answer for each question corresponds to the row of answer circles for that question. Place the key on the answer sheet so that the 3 appears in the box for question 1, and the 4 appears in the box for question 27. Allow 2 credits for each correct answer.

LIST OF CORRECT ANSWERS

(1) 3   (12) 1   (23) 1
(2) 2   (13) 1   (24) 1
(3) 3   (14) 3   (25) 1
(4) 1   (15) 3   (26) 4
(5) 4   (16) 2   (27) 4
(6) 3   (17) 1
(7) 3   (18) 1
(8) 4   (19) 1
(9) 4   (20) 3
(10) 1  (21) 2
(11) 2  (22) 3