The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, August 16, 2000 — 8:30 to 11:30 a.m., only

Notice . . .
Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
1. In the accompanying diagram, line \( a \) is parallel to line \( b \) and line \( c \) is a transversal. If \( m\angle 1 = 2x \) and \( m\angle 2 = 5x - 54 \), what is the value of \( x \)?

2. Find the value of \( M \odot (E \odot T) \) in the system defined below.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\odot & M & E & T & S \\
\hline
M & T & S & M & E \\
E & S & M & E & T \\
T & M & E & T & S \\
S & E & T & S & M \\
\hline
\end{array}
\]

3. In \( \triangle REC \), \( m\angle E = 55 \) and \( m\angle R = 65 \). Which side of \( \triangle REC \) is the shortest?

4. In the accompanying diagram of \( \triangle ABC \), \( AB \) is extended through \( B \) to \( D \). If \( m\angle CBD = 3x + 20 \), \( m\angle A = x \), and \( m\angle ACB = x + 60 \), find \( x \).

5. If operation \( \boxplus \) is defined as \( a \boxplus b = \frac{a}{b} + 3 \), \( b \uparrow 0 \), find the value of \( 3 \boxplus 6 \).

6. In \( \triangle ABC \), the midpoint of \( \overline{AC} \) is \( R \), the midpoint of \( \overline{CB} \) is \( S \), and the midpoint of \( \overline{AB} \) is \( T \). If \( AC = 3 \), \( CB = 4 \), and \( AB = 5 \), what is the perimeter of \( \triangle RST \)?

7. Solve this system of equations for the positive value of \( y \).

\[
\begin{align*}
x &= 2y \\
x + y^2 &= 8
\end{align*}
\]

8. The lengths of the sides of a triangle are 7, 8, and 10. If the length of the longest side of a similar triangle is 25, what is the length of the shortest side of this triangle?

9. In the accompanying diagram, altitude \( \overline{CD} \) is drawn to the hypotenuse of right triangle \( ABC \). If \( AD = 9 \) and \( AB = 13 \), find \( CD \).

10. If a dilation maps \((-3,2)\) to \((x,8)\), what is the value of \( x \)?

11. The coordinates of the turning point of the graph of the equation \( y = 2x^2 - 4x + 6 \) are \((1,k)\). What is the value of \( k \)?

12. If the number of degrees in a base angle of an isosceles triangle is four times the number of degrees in the vertex angle, what is the number of degrees in a base angle of the triangle?
13 What is the midpoint of the line segment whose endpoints are (7,–4) and (–3,–2)?

Directions (14–34): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

14 Which law of logic is represented in this argument?

\[ \sim a \rightarrow b \\
\sim a \\
\therefore b \]

(1) DeMorgan’s Law
(2) Law of Detachment
(3) Law of Disjunctive Inference
(4) Law of Contrapositive

15 If the point (5,1) is reflected in the y-axis, the image is

(1) (–5,1) (3) (5,1)
(2) (–5,–1) (4) (5,–1)

16 What is the total number of different eight-letter permutations that can be formed from the letters in the word “LOLLIPOP”?

(1) \( \frac{8!}{2!3!} \) (3) \( \frac{8!}{2!2!3!} \)
(2) \( \frac{8!}{7!} \) (4) 8!

17 Given the conditional statement \( p \rightarrow q \), which statement is true?

(1) The inverse is \( p \rightarrow \sim q \).
(2) The converse is \( q \rightarrow p \).
(3) The contrapositive is \( \sim p \rightarrow \sim q \).
(4) The inverse of the converse is \( \sim q \rightarrow p \).

18 Which statement is the negation of “I work or I do not have money”?

(1) I do not work or I have money.
(2) I do not work and I have money.
(3) I do not work and I do not have money.
(4) I work and I have money.

19 The distance between coordinates \( D(–4,–3) \) and \( E(5,9) \) is

(1) \( \sqrt{37} \) (3) 12
(2) \( \sqrt{63} \) (4) 15

20 In the accompanying diagram of \( \triangle ABD \), \( C \) is a point on \( AD \), \( BC \) is drawn, \( m\angle A = 65 \), \( m\angle BCD = 135 \), and \( m\angle CBD = 20 \).

Which statement must be true?

(1) \( BC \perp AD \) (3) \( AB \equiv BD \)
(2) \( AC \equiv CD \) (4) \( AB \perp BD \)

21 Which set may be the lengths of the sides of an isosceles triangle?

(1) \{1,1,2\} (3) \{5,12,13\}
(2) \{3,3,8\} (4) \{4,4,6\}

22 Which statement is true about all parallelograms?

(1) The diagonals are congruent.
(2) The area is the product of two adjacent sides.
(3) The opposite angles are congruent.
(4) The diagonals are perpendicular to each other.

23 The solution set of \( 10x^2 - 48x + 32 = 0 \) is

(1) \{–8,4\} (3) \{4, \frac{4}{5}\}
(2) \{4, \frac{1}{5}\} (4) \{–4, \frac{4}{5}\}

24 The sum of \( \frac{2}{x} + \frac{2}{y} \) is

(1) \( \frac{2}{x+y} \) (3) \( \frac{4}{xy} \)
(2) \( \frac{4}{x+y} \) (4) \( \frac{2y+2x}{xy} \)
25 Which expression is equal to 15?
(1) \(6C_4\)  
(2) \(6P_4\)  
(3) \(15C_{15}\)  
(4) \(6P_2\)

26 If a side of a square has length 14, the length of a diagonal of the square is
(1) 14  
(2) \(2\sqrt{14}\)  
(3) \(14\sqrt{2}\)  
(4) 28

27 If the slope of a straight line is 0, the graph of this line may pass through Quadrants
(1) I and II  
(2) I and III  
(3) I and IV  
(4) II and IV

28 In \(\triangle ABC\), \(m\angle A = 25\) and \(m\angle C = 90\). Which ratio represents \(\tan 65^\circ\)?
(1) \(\frac{AC}{AB}\)  
(2) \(\frac{AC}{BC}\)  
(3) \(\frac{AB}{AC}\)  
(4) \(\frac{BC}{AC}\)

29 What is the equation of a circle whose center is (2,−3) and whose radius is 4?
(1) \((x + 2)^2 + (y - 3)^2 = 4\)  
(2) \((x - 2)^2 + (y + 3)^2 = 2\)  
(3) \((x + 2)^2 + (y - 3)^2 = 16\)  
(4) \((x - 2)^2 + (y + 3)^2 = 16\)

30 Which equation represents a line parallel to the line whose equation is \(2y = 3x + 6^2\)
(1) \(3y = 2x + 6\)  
(2) \(2y = -3x + 6\)  
(3) \(y = \frac{2}{3}x + 1\)  
(4) \(y = \frac{3}{2}x - 4\)

31 Lines \(\ell\) and \(m\) are parallel lines 8 centimeters apart, and point \(P\) is on line \(m\). What is the total number of points that are equidistant from lines \(\ell\) and \(m\) and 5 centimeters from \(P\)?
(1) 1  
(2) 2  
(3) 0  
(4) 4

32 The perimeter of a rhombus is 60. If the length of its longer diagonal measures 24, the length of the shorter diagonal is
(1) 9  
(2) 15  
(3) 18  
(4) 20

33 The lengths of the bases of an isosceles trapezoid are 6 centimeters and 12 centimeters. If the length of each leg is 5 centimeters, what is the area of the trapezoid?
(1) 18 cm²  
(2) 36 cm²  
(3) 45 cm²  
(4) 90 cm²

34 If each interior angle of a regular polygon measures 135°, the polygon must be
(1) an octagon  
(2) a decagon  
(3) a hexagon  
(4) a pentagon

Directions (35): Leave all construction lines on the answer sheet.

35 On the answer sheet, construct the perpendicular bisector of segment \(XY\).
Answers to the following questions are to be written on paper provided by the school.

Part II

Answer three questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown.

36 Find the area of pentagon CANDY with vertices C(–6,8), A(3,8), N(6,–2), D(–4,–1), and Y(–7,4).

37 Answer a, b, and c for all values of x for which these expressions are defined.
   a Simplify: \( \frac{4x^2 - 9}{2x^2 - x - 6} \cdot \frac{4x - 8}{2x - 3} \) [4]
   b Express as a single fraction in lowest terms:
      \( \frac{1}{x + 2} + \frac{x}{2x + 4} \) [3]
   c Solve for x: \( \frac{2x}{5} - \frac{x - 2}{10} = 2 \) [3]

38 a Draw the locus of points 6 units from the origin and label it with its equation. [3]
   b Draw the locus of points 6 units from the x-axis and label it with its equations. [3]
   c Following the rule \((x, y) \rightarrow (x + 6, y)\), graph the transformation of the locus in part a, and label the graph with its equation. [4]

39 a Draw and label the graph of the equation \( y = 2x^2 - 8x + 1 \), including all values of x such that \(-1 \leq x \leq 5\). [6]
   b Using an algebraic method, find the roots of \( 2x^2 - 8x + 1 = 0 \) to the nearest tenth. [4]

40 A jar contains yellow marbles, red marbles, and blue marbles. The number of red marbles is three less than twice the number of blue marbles. The number of yellow marbles is one more than seven times the number of blue marbles. The probability of selecting a yellow marble is \( \frac{5}{4} \).
   a Find the number of marbles of each color in the jar. [5]
   b Three marbles are taken from the jar without replacement.
      (1) What is the total number of different three-marble selections that can be made? [2]
      (2) What is the probability that the three marbles selected will be one of each color? [3]
Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41 Given: Jim drives a car or Jim takes a bus.
   If Jim takes a bus, then Jim carries his bus pass.
   Jim does not carry his bus pass.
   If Jim drives a car, then Jim buys gasoline.
   If Jim buys gasoline, then Jim has a job.

   Let C represent: “Jim drives a car.”
   Let B represent: “Jim takes a bus.”
   Let P represent: “Jim carries his bus pass.”
   Let G represent: “Jim buys gasoline.”
   Let J represent: “Jim has a job.”

   Prove: Jim has a job. [10]

42 Quadrilateral QUAD has coordinates Q(−a,0), U(3a,0), A(2a,2a), and D(0,2a).
   Using coordinate geometry, prove that quadrilateral QUAD is an isosceles trapezoid. [10]
Your answers to Part I should be recorded on this answer sheet.

Part I
Answer 30 questions from this part.

1 .......................... 11 .......................... 21 .......................... 31 ..........................
2 .......................... 12 .......................... 22 .......................... 32 ..........................
3 .......................... 13 .......................... 23 .......................... 33 ..........................
4 .......................... 14 .......................... 24 .......................... 34 ..........................
5 .......................... 15 .......................... 25 .......................... 35 Answer question 35 on the other side of this sheet.
6 .......................... 16 .......................... 26 ..........................
7 .......................... 17 .......................... 27 ..........................
8 .......................... 18 .......................... 28 ..........................
9 .......................... 19 .......................... 29 ..........................
10 .......................... 20 .......................... 30 ..........................
Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature
FOR TEACHERS ONLY

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SCORING KEY

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the student’s work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 14–34, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 18  
(2) S  
(3) \( \overline{RC} \)  
(4) 40  
(5) 3 \( \frac{1}{2} \)  
(6) 6  
(7) 2  
(8) 17.5  
(9) 6  
(10) –12

(11) 4  
(12) 80  
(13) (2,–3)  
(14) 2  
(15) 1  
(16) 3  
(17) 2  
(18) 2  
(19) 4

(21) 4  
(22) 3  
(23) 3  
(24) 4  
(25) 1  
(26) 3  
(27) 1  
(28) 2  
(29) 4

(31) 2  
(32) 3  
(33) 2  
(34) 1  
(35) construction  
(30) 4

[OVER]
Part II

Please refer to the Department’s publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) 97.5  [10]

(37)  a  4  [4]
      b  1/2  [3]
      c  6  [3]

(38)  a  \(x^2 + y^2 = 36\)  [3]
      b  \(y = 6, y = -6\)  [3]
      c  \((x - 6)^2 + y^2 = 36\)  [4]

(39)  b  0.1, 3.9  [4]

(40)  a  5 blue marbles
      b  7 red marbles  [5]
      c  36 yellow marbles

b  (1) 17,296  [2]

(2) \[
\frac{1260}{17,296} \]

As a reminder . . .

Regents examinations based on the Sequential Mathematics, Course II, syllabus will not be offered after January 2003.

Regents examinations based on the Sequential Mathematics, Course I, syllabus will not be offered after January 2002.