The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, June 23, 1999 — 1:15 to 4:15 p.m., only

Notice . . .
Scientific calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of \( \pi \) or in radical form.  

1. If the operation \( \bullet \) is defined as \( a \bullet b = 2a - b^2 \), evaluate \( 3 \bullet 2 \).

2. In the diagram below, \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) intersect at \( E \), \( m\angle AEC = 6x - 20 \), and \( m\angle BED = 4x + 10 \). Find the value of \( x \).

3. In isosceles triangle \( ABC \), \( \overrightarrow{CA} \equiv \overrightarrow{CB} \). If side \( \overrightarrow{CA} \) is extended through \( A \) to \( F \) and \( \overrightarrow{FB} \) is drawn, what is the longest side of \( \triangle FAB \)?

4. The ratio of two complementary angles is 7:2. What is the measure, in degrees, of the larger angle?

5. In the accompanying diagram of \( \triangle ABC \), \( \overrightarrow{DE} \parallel \overrightarrow{AB} \), \( DE = 8 \), \( CD = 12 \), and \( DA = 3 \). Find the length of \( \overrightarrow{AB} \).

6. Solve for \( x \): \[ \frac{x}{6} + \frac{2x}{3} = 5 \]

7. After a dilation with respect to the origin, the image of point \( A(3,4) \) is \( A'(9,12) \). What are the coordinates of the image of point \( B(2,7) \) after the same dilation?

8. In the accompanying diagram, \( \overrightarrow{AB} \) is parallel to \( \overrightarrow{CD} \); transversal \( \overrightarrow{KL} \) intersects \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) at \( E \) and \( F \), respectively; \( m\angle BEF = 3x + 40 \); and \( m\angle DFL = 8x - 10 \). Find \( m\angle CFL \).

9. Points \( R, S \), and \( T \) are the midpoints of the sides of a triangle whose sides have lengths 14, 18, and 20. Find the perimeter of \( \triangle RST \).

10. Solve the following system of equations for the positive value of \( x \):
   
   \[
   \begin{align*}
   y &= 5x + 14 \\
   y &= x^2
   \end{align*}
   \]

11. In the accompanying diagram of right triangle \( ABC \), \( CD \) is the altitude to hypotenuse \( \overrightarrow{AB} \), \( CD = 6 \), and \( DB = 4 \). Find the length of \( AD \).
12 In which quadrant does the image of $A(3,-1)$ lie after the translation $(x,y) \rightarrow (x+4, y-2)$?

**Directions (13–34): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.**

13 Given the true statements:
   "If Peter is a rabbit, then Felix is a cat."
   "Felix is not a cat."
Which statement must also be true?
   (1) Felix is a cat.
   (2) Felix is not a rabbit.
   (3) Peter is a cat.
   (4) Peter is not a rabbit.

14 Which statement is logically equivalent to $\sim (p \lor \sim r)$?
   (1) $p \land r$
   (2) $\sim p \lor r$
   (3) $\sim p \land r$
   (4) $\sim p \lor \sim r$

15 The sum of $\frac{4}{5x}$ and $\frac{5}{4x}$ is equivalent to
   (1) $x$
   (2) $41x^2$
   (3) $\frac{41}{9x}$
   (4) $\frac{41}{20x}$

16 What are the coordinates of the image of point (−2,6) after a reflection in the y-axis?
   (1) (2,6)
   (2) (6,−2)
   (3) (2,−6)
   (4) (−2,−6)

17 In isosceles trapezoid $ABCD$, $AD = BC$. What is $m\angle A + m\angle C$?
   (1) 45
   (2) 90
   (3) 180
   (4) 360

18 If the turning point of a parabola is (4,−3) and the axis of symmetry is parallel to the y-axis, then the equation of the axis of symmetry is
   (1) $x = -3$
   (2) $y = -3$
   (3) $x = 4$
   (4) $y = 4$

19 In the accompanying diagram of $\triangle ABC$, which expression can be used to determine $m\angle A$?

   ![Diagram of triangle ABC with sides labeled 5, 12, 13, angles labeled A, B, C]

   (1) $\sin A = \frac{12}{13}$
   (2) $\cos A = \frac{12}{5}$
   (3) $\cos A = \frac{5}{13}$
   (4) $\tan A = \frac{5}{12}$

20 What is the length of the line segment joining points $B(-7,2)$ and $E(1,8)$?
   (1) $\sqrt{72}$
   (2) 10
   (3) $\sqrt{164}$
   (4) $12$

21 Which equation represents the locus of points equidistant from the lines whose equations are $y = 3x + 8$ and $y = 3x - 6$?
   (1) $y = 3x - 1$
   (2) $y = 3x + 1$
   (3) $y = 3x - 4$
   (4) $y = 3x + 4$

22 If the diagonals of a rhombus are 12 and 16, then a side of the rhombus will measure
   (1) 10
   (2) 12
   (3) 16
   (4) 20

23 How many committees of three students can be formed from a class of nine students?
   (1) $9C_3$
   (2) $9P_3$
   (3) $\frac{9!}{3!}$
   (4) $(9 - 3)!$

24 What is the total number of different six-letter arrangements that can be formed from the letters in the word "POWWow"?
   (1) 720
   (2) 60
   (3) 20
   (4) 6

25 If the lengths of two sides of a triangle are 2 and 5, the length of the third side could be
   (1) 1
   (2) 2
   (3) 6
   (4) 7
Part III

Answer one question from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [10]

41. Given: If Patty plays soccer, then Carl plays football.
   If Carl plays football and Dana plays field hockey, then Scott does not play golf.
   If Frank does not play volleyball, then Scott plays golf.
   Dana plays field hockey.
   Frank does not play volleyball.

Let $P$ represent: “Patty plays soccer.”
Let $C$ represent: “Carl plays football.”
Let $S$ represent: “Scott plays golf.”
Let $F$ represent: “Frank plays volleyball.”
Let $D$ represent: “Dana plays field hockey.”

Prove: Patty does not play soccer. [10]

42. Given: rhombus $RSTV$, $\overline{VTX}$, $\overline{STW}$, $\overline{SX}$, $\overline{VW}$, and $\angle RSX \equiv \angle RVW$.

Prove: $\overline{TX} \equiv \overline{TW}$
Part II

Answer three questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [30]

36 Solve the following system of equations graphically and check:
\[
\begin{align*}
(x + 2)^2 + (y - 1)^2 &= 16 \quad [8, 2] \\
x - y &= 1
\end{align*}
\]

37 a Find, to the nearest tenth, all values of \( x \) for which the expression is defined:
\[
\frac{x - 3}{5} = \frac{2}{x + 2} \quad [5]
\]
b A committee of five is to be chosen from 6 freshmen and 8 sophomores. What is the probability that the committee will include 2 freshmen and 3 sophomores? [5]

38 The coordinates of the endpoints of line segment \( ME \) are \( M(-4, -1) \) and \( E(5, 4) \).

a On graph paper, draw and label the graph of \( ME \). [11]
b Segment \( ST \) is the image of \( ME \) after the transformation \( (x, y) \rightarrow (x + 2, y - 6) \), where \( S \) is the image of \( M \), and \( T \) is the image of \( E \). On the same set of axes, draw and state the coordinates of the endpoints of the graph of \( ST \). [3]
c Is quadrilateral \( METS \) a parallelogram? Justify your answer using coordinate geometry. [15]

39 In the tables below, the elements \( S, A, L, E \) and the operations \( \ast \) and \( \# \) are defined.

<table>
<thead>
<tr>
<th>( \ast )</th>
<th>S</th>
<th>A</th>
<th>L</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>A</td>
<td>S</td>
<td>E</td>
<td>L</td>
</tr>
<tr>
<td>A</td>
<td>S</td>
<td>A</td>
<td>L</td>
<td>E</td>
</tr>
<tr>
<td>L</td>
<td>E</td>
<td>L</td>
<td>A</td>
<td>S</td>
</tr>
<tr>
<td>E</td>
<td>L</td>
<td>E</td>
<td>S</td>
<td>A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#</th>
<th>S</th>
<th>A</th>
<th>L</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>A</td>
<td>L</td>
<td>E</td>
<td>S</td>
</tr>
<tr>
<td>A</td>
<td>L</td>
<td>E</td>
<td>S</td>
<td>A</td>
</tr>
<tr>
<td>L</td>
<td>E</td>
<td>S</td>
<td>A</td>
<td>L</td>
</tr>
<tr>
<td>E</td>
<td>L</td>
<td>E</td>
<td>S</td>
<td>A</td>
</tr>
</tbody>
</table>

a What is the inverse of \( L \) under \( \# \)? [2]
b Evaluate: \( (A \ast L) \# (E \ast S) \) [2]
c Solve for \( x \): \( (x \# L) \ast L = A \) [2]
d Find all values of \( x \) such that \( x \ast x = x \# x \). [4]

40 Trapezoid \( ABCD \) has coordinates \( A(0, 9), B(12, 9), C(8, 4), \) and \( D(0, 4) \).

a Using coordinate geometry, show that \( \overline{AD} \) is not parallel to \( \overline{BC} \). [3]
b Find the area of trapezoid \( ABCD \). [2]
c Find the perimeter of \( ABCD \) to the nearest integer. [5]
d Find \( m \angle ABC \) to the nearest degree. [2]

GO RIGHT ON TO THE NEXT PAGE. 

Math.--Course II--June '99 [5] [OVER]
26 Points $A(-1,3)$ and $B(4,1)$ are endpoints of a diameter of a circle. The coordinates of the center of this circle are

(1) $(3,4)$  
(2) $(2,2)$  
(3) $(1,5,2)$  
(4) $(0,3,5)$

27 A parallelogram must be a rectangle if its diagonals

(1) are congruent
(2) bisect each other
(3) bisect the angles through which they pass
(4) are perpendicular to each other

28 In the diagram below of right triangle $BAC$, $m\angle A = 90$, $m\angle B = 45$, and $AC = 8$.

![Diagram of triangle BAC with coordinates and angle 45°](http://example.com/diagram)

What is the length of $BC$?

(1) $8\sqrt{3}$
(2) $8\sqrt{2}$
(3) $4\sqrt{2}$
(4) $16\sqrt{2}$

29 What is the equation of the line that is perpendicular to the line $y - 2x = 4$ and passes through point $(2,4)$?

(1) $y = \frac{1}{2}x + 4$
(2) $y = -\frac{1}{2}x + 5$
(3) $y = \frac{1}{2}x + 5$
(4) $y = -2x + 5$

30 What is the slope of the line that passes through the points $(-2,4)$ and $(8,-1)$?

(1) $-\frac{1}{2}$
(2) $\frac{1}{2}$
(3) $\frac{2}{1}$
(4) $-\frac{2}{1}$

31 Under which operation is the set of odd integers closed?

(1) addition
(2) subtraction
(3) multiplication
(4) division

32 The length of a diagonal of a square is 8. What is the area of the square?

(1) 8
(2) 16
(3) 24
(4) 32

33 Which fraction is expressed in simplest form?

(1) $\frac{x-1}{x^2-1}$
(2) $\frac{x-1}{x^3-2x+1}$
(3) $\frac{x+1}{x^2-1}$
(4) $\frac{x+1}{x^2+1}$

34 In regular pentagon $ABCDE$, the measure of an exterior angle at $A$ is

(1) $36^\circ$
(2) $60^\circ$
(3) $72^\circ$
(4) $108^\circ$

*Directions (35): Leave all construction lines on the answer sheet.*

35 *On the answer sheet,* construct the bisector of angle $C$.  

---

Math. Course II—June ’99
ANSWER SHEET

Pupil .................................................. Sex: ☐ Male ☐ Female Grade ..............
Teacher .................................................. School ......................................

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>21</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>22</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>23</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>24</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>19</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Answer question 35 on the other side of this sheet.
Your answers for Part II and Part III should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

____________________________
Signature

Math.-Course II-June '99
FOR TEACHERS ONLY

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Wednesday, June 23, 1999 — 1:15 to 4:15 p.m., only

SCORING KEY

Use only red ink or red pencil in rating Regents papers. Do not attempt to correct the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 13–34, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>FFB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(6,21)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>IV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>construction</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[OVER]
Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) \(-2, -3\), \((2, 1)\) \(\text{Check} \quad [8]\)

(37) \(a\) -3.5, 4.5 \(\quad [5]\)
\(b\) \(\frac{840}{2002} \quad [5]\)

(38) \(b\) \(S(-2,-7), T(7,-2)\) \(\quad [3]\)
\(c\) Yes \(\quad [1]\)

(39) \(a\) \(S \quad [2]\)
\(b\) \(A \quad [2]\)
\(c\) \(E \quad [2]\)
\(d\) \(S, L \quad [4]\)

(40) \(b\) \(50 \quad [2]\)
\(c\) \(31 \quad [3]\)
\(d\) \(51 \quad [2]\)

Justification \(\quad [8]\)
FOR TEACHERS ONLY

ERRATA SHEET

REGENTS HIGH SCHOOL EXAMINATION

Sequential Mathematics, Course II

Wednesday, June 23, 1999—1:15 to 4:15 p.m., only

At the start of the Course II examination, please tell students that there is a spelling error in question 38, on page 5 of the examination booklet. The correction should also be written on the chalkboard, if one is available.

In part c of question 38, the word printed as "quadrilateral" should be "quadrilateral."